Explore

FRACTIONS & RATIO

Struggling students will benefit if you provide a context for their fraction work by drawing the connection between division and fractions. Money and music are also great real world examples of fractions. Keep in mind that the second part of the lesson will explore the similar relationships in ratios, which are expressed as part to part. Definitions are important to this lesson, so you may want to track the vocabulary words as you build the lesson on your board.

Explore

FRACTIONS & RATIO

A fraction is a part of a whole number. Fractions show up in many aspects of our lives. Money, for example, is largely about fractions. A quarter is $\frac{1}{4}$ of a whole dollar. But what if I have 2 quarters? We typically refer to this as 50 cents, but it is also $\frac{1}{2}$ of that dollar. One way to approach fractions is to simply think of them as division. If we have one dollar and divide it in 2 pieces, each piece represents one half (or $\frac{1}{2}$) of that dollar. In order to know how much money we have, we need to be able to add, subtract, multiply, and divide fractions. Fractions are also common on the ACT, so it's important that we thoroughly understand how they work and how to manipulate them.

At the end of this lesson, you will be able to

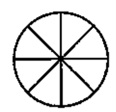
a. add, subtract, multiply, and divide fractions

b. solve problems using ratios and proportions

Fraction Basics

As we already mentioned, a fraction is a *part to whole* relationship. The fraction itself is made up of two pieces. The top of the fraction is called the **numerator,** and represents the *part* that we are given. The bottom of the fraction is called the **denominator.** It represents the *whole.*

Let's look at an example. The pizza below is cut into 8 equal slices. If we want to describe a slice of the pizza as a fraction, we can put the 1 slice (or part) in the numerator of the fraction and the 8 total slices (or whole) in the denominator. If we want to talk about the entire pizza, we can represent that by the putting the parts (8 slices) over the whole (8 slices). Notice that when we do the division, we get one – the whole pizza.



One Slice: $\frac{1}{8}$ Whole Pizza: $\frac{8}{8}$

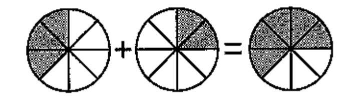
Sometimes on the ACT, we will need to reduce fractions to their simplest form so that we can match our answer to one of the answer choices. To do this, just look for a factor that the numerator and denominator have in common. For example, if we are given the fraction 6_8 , we might notice that both 6 and 8 are divisible by 2. When we divide 6 by 2, we get 3. When 8 is divided by 2, the answer is 4. Therefore:

$$\frac{6}{8}=\frac{3}{4}$$

Since 3 and 4 do not share any common factors, we know that we have reduced our fraction to its simplest possible form.

Addition and Subtraction

Sometimes, the ACT will ask us to add or subtract fractions. What if you had 3 slices of that pizza on your plate, and wanted to add two more slices? What fraction of the pizza would you have then? The easiest fractions to add and subtract are ones that have the same number in the denominator. For these, we simply leave the denominator alone and add or subtract the numbers in the numerators. Let's look at how that would work for our pizza problem:



$$\frac{3\ slices\ of\ pizza}{8\ total\ slices}+\frac{2\ slices\ of\ pizza}{8\ total\ slices}=\frac{3+2}{8}=\frac{5}{8}$$

We would then have $\frac{5}{8}$ of the entire pizza on our plate.

$$\frac{5}{9}+\frac{2}{9}=\frac{5+2}{9}=\frac{7}{9}$$

$$\frac{5}{11}-\frac{3}{11}=\frac{5-3}{11}=\frac{2}{11}$$

Let's look at a few examples where the denominators in the two fractions are not the same: When you don't have common denominators, use the **Bowtie Method.** Consider this example.

$$\frac{2}{7}+\frac{3}{5}$$

First, draw two arrows pointing up and across, like this:

$$\frac{2}{7} \times \frac{3}{5}$$

Now multiply across each arrow, and write the result above the numerator to which the arrow is pointing.

$$\overset{10}{\frac{2}{7}} \times \overset{21}{\frac{3}{5}}$$

The problem asked us to add the fractions, so add 10 to 21 to get 31. This will be the numerator of our answer.

$$\overset{10}{\frac{2}{7}} \times \overset{21}{\frac{3}{5}} = \frac{10+21}{\ } = \frac{31}{\ }$$

Addition and Subtraction

If students struggle with the rules and definitions, start with an analogy. You could talk about the number of girls in the class as a fraction. You could then add the number of boys with glasses as a fraction to the number of boys with glasses as a fraction to show addition. This sort of example may also work in your favor when you get to ratios.

Next, we simply need to multiply our denominators:

$$\overset{10}{\frac{2}{7}}\quad\overset{21}{\frac{3}{5}} = \frac{10+21}{35} = \frac{31}{35}$$

Subtraction works exactly the same way.

$$\overset{8}{\frac{4}{5}}\quad\overset{5}{\frac{1}{2}} = \frac{8-5}{10} = \frac{3}{10}$$

Multiplication and Division

Multiplying and dividing fractions actually requires fewer steps than adding or subtracting them. First, let's look at multiplication. How would we multiply the following fraction?

$$\frac{14}{15}\times\frac{5}{6}$$

To solve this problem, we can just multiply the numerators and denominators separately, and then reduce.

$$\frac{14}{15}\times\frac{5}{6}=\frac{70}{90}=\frac{7}{9}$$

What if we didn't want to deal with really big numbers? We could also reduce the fractions before we multiply. Neither nor can be reduced on its own; however, when we multiply fractions (and only when we multiply), we can cancel diagonally—or reduce. In order words, we can reduce the numerator of one fraction by a common factor of the denominator of the other fraction. In this case, we can reduce the 14 and 6 by 2 and the 5 and 15 by 5. This leaves us with

$$\frac{14}{15} \text{ x } \frac{5}{6} = \frac{7}{3} \text{ x } \frac{1}{3} = \frac{7}{9}$$

Reducing before multiplying is usually easier, since in most cases it's simpler to work with fractions when the numbers are still small.

Dividing fractions is just as easy. It just adds an extra, simple step: taking the reciprocal of the second fraction. Remember: When dividing, don't ask why, flip the second and multiply. Let's try one:

$$\frac{3}{5} \div \frac{2}{3}$$

We just need to multiply the first fraction by the reciprocal of the second fraction. That's all there is to it!

$$\frac{3}{5} \text{ x } \frac{3}{2} = \frac{9}{10}$$

Sneak Peek

3. If $m = \frac{2}{3}$ and $n = \frac{1}{5}$, what is the value of $\frac{1}{3}(m+n)$?

A. $\frac{3}{8}$

B. $\frac{3}{11}$

C. $\frac{11}{45}$

D. $\frac{13}{45}$

E. $\frac{17}{45}$

Ratios and Proportions

Fractions are used to compare parts to a whole. For example, if a pet shop has 10 animals, 7 of which are dogs, then $\frac{7}{10}$ of the animals are dogs.

Ratios, however, are used to compare parts to parts. If, in that same pet shop, all animals are either dogs or cats, then the ratio of dogs to cats is 7:3, since there are $10 - 7 = 3$ cats.

When we have two ratios set equal to each other, this is called a **proportion.** This can be done in cases in which we have a ratio between two items and an actual number of one of those items. For example, if we know that in a bag of marbles, there are 3 red marbles for every 5 blue marbles and that there are 12 red marbles in the bag, we can determine the number of blue marbles. Set what we know equal to what we want to know.

$$\frac{3\ \textit{red marbles}}{5\ \textit{blue marbles}} = \frac{12\ \textit{red marbles}}{x\ \textit{blue marbles}}$$

Sneak Peek

This Sneak Peek involves Algebra, so you may want to do a quick refresher of how to work with variables or, for students who aren't prepared to tackle Algebra just yet, come back to it later.

Ratios and Proportions

Draw the distinction between ratios and fractions here. A lot of students are familiar with ratios expressed as x to y, but are not familiar with ratios expressed as fractions. The SAT tests proportions and ratios regularly, so it is an important skill to develop for the test.