Explore

ANGLES

Ask students what *methodically* means. This is very important to emphasize because students will need a consistent methodical approach to solving ACT Geometry problems instead of relying on "seeing" various angles and shapes. Having a good starting plan will help jumpstart the problem-solving process for many students.

Naming and Categorizing Angles

Stress how important this skill will be to attack even the most basic of ACT Geometry problems. Draw and label each respective angle on the board so that students have another visual imprint of what each angle is. Repeatedly refer to these shapes on the board any time each of these angles is mentioned throughout the duration of the lesson. Associating everyday objects with each of the different angles can also help students more effectively learn all of these angles.

Make sure it's clear to the students that the center letter is the vertex of the angle. Assure them that they can order the letters either way they want, but the vertex must be the middle letter.

Explore

ANGLES

Knowing how to classify angles can help you understand how to solve an ACT geometry problem. No matter how complex the figure might look, you will be able to methodically solve the problem by breaking down the complex figure into smaller, more manageable parts.

At the end of lesson, you will be able to

a. categorize and name angles

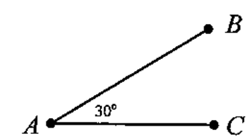
b. understand the vocabulary of angles

c. understand how parallel lines and intersecting lines form specific types of angles

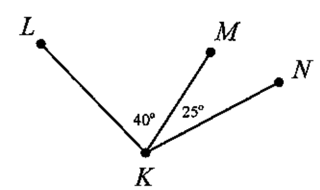
d. understand how angles are tested on the ACT

Naming and Categorizing Angles

An angle is formed by two intersecting lines. The size of the angle determines what kind of angle it is. An acute angle is smaller than 90°. The vertex is where the two lines meet and is usually how the angle is named. The below angle would be called "angle *A*" since the vertex is at Point *A*. On the ACT, the angle would be formally called "angle *BAC*" or, more commonly, symbolized as $\angle BAC$.



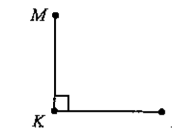
Now, what if we have multiple lines? No problem. $\angle LKM$ is 40°, and $\angle MKN$ is 25°. Together, they make up $\angle LKN$. We can find the measure of $\angle LKN$ by adding 40° and 25° to get 65°.



Students will need to be extremely familiar with the perpendicular symbol, □, which will be used frequently in chapters in which shapes and solids are examined more closely. You may therefore want to write this relationship up on the board.

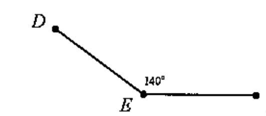
If students are unfamiliar with line nomenclature, explain to them that a line is defined by two end points with a double-headed arrow or horizontal line above the letters.

A **right** angle measures exactly 90°. On the ACT, a right angle will always be denoted by a little square that represents a 90° angle. The figure below represents right $\angle MKN$. Whenever you see the little square, it will always mean that the angle is a right angle; however, if you DON'T see the little square, you CANNOT assume that the angle is a right angle – no matter how "right" it may look.

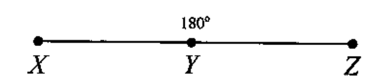


When two lines intersect at exactly 90°, we say that these lines are **perpendicular** to each other. Again using the previous figure, we denote perpendicular lines by writing $\overline{MK} \perp \overline{KN}$.

An **obtuse** angle is greater than 90°. $\angle DEF$, as shown below, is an example of an obtuse angle.



A straight angle measures *exactly* 180°. $\angle XYZ$, shown below, is an example of a straight angle but would simply be called a "straight line" on the ACT. Thus, if smaller angles can be added together to equal 180°, they would collectively make a straight line. Since this concept is very often tested on the ACT, recognizing this will be very useful in helping you to attack problems.



Manipulating Angles

If your students clearly understand the concepts on this page, then you may want to let them practice the problems on their own before reviewing each one as a class.

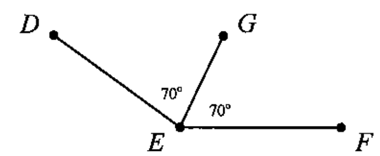
This might be a good problem to put on the board and walk through with students. Draw $\angle DEF$ without FG on the board and label it as a 140o angle. Before bisecting it, ask your students what type of angle $\angle DEF$ is (obtuse). You could also ask for alternative ways to name this angle just to reinforce that angles can be named a variety of ways – as long as the angle's vertex is the middle letter. Once you do this, draw the angle bisector and ask the students how many degrees each of the smaller angles now is.

Sneak Peak

(E) 130°. Since $\angle$DBA equals 50°, $\angle$ DBC must equal 130° to form the 180° of $\angle$ABC.

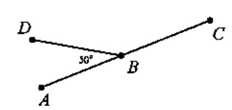
Manipulating Angles

An angle that is **bisected** is an angle that has been split into two, equal smaller angles. The figure below is the obtuse angle from the previous section that has been bisected into two smaller equal angles, $\angle DEG$ and $\angle GEF$.



Since $\angle DEG$ and $\angle GEF$ are equal, they would be called congruent angles. On the ACT, congruent angles are denoted in the following way: $\angle DEG \cong \angle GEF$.

Sneak Peek



8. Straight line segments $\overline{AC}$ and $\overline{BD}$ intersect at point B as shown. *What is the measure of $\angle DBC$?*

A. 40°

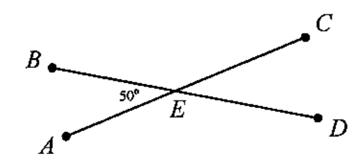
B. 60°

C. 70°

D. 100°

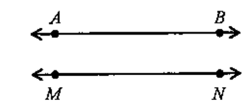
E. 130°

Angles can also be created when two lines intersect. What angles are created by the intersecting lines in the figure below?

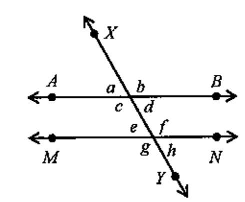


That's right. We have $\angle AEB$, $\angle BEC$, $\angle CED$, and $\angle AED$ that are formed by intersecting lines AEC and BED. $\angle AEB$ and $\angle CED$ are known as vertical, or opposite, angles. Vertical angles are always congruent, so whenever you see a problem like this on the ACT, you will instantly know that $\angle AEB \cong \angle CED$ and $\angle BEC \cong \angle AED$. *This concept appears quite often on the ACT so thoroughly understanding it will be extremely beneficial. If $\angle AEB$ is 50°, as shown above, what would the measures of the other angles be?*

Now, what if two lines *didn't* intersect? That is, no matter how far you extend two lines, they would never cross each other? These lines are called **parallel**, and parallel lines make up one of the most powerful concepts in ACT Geometry. The figure below shows parallel lines $\overleftrightarrow{AB}$ and $\overline{MN}$, symbolized as $\overleftrightarrow{AB} \parallel \overline{MN}$.



A line that intersects both of these parallel lines is called a **transversal** (also known as a **transverse**). In the below figure, the transversal creates eight angles when it cuts through the parallel lines, and these eight angles have special relationships with each other.



Mastery of parallel lines is essential to success in ACT Geometry, so this concept is an extremely important one. Draw the parallel lines on the board and repeatedly refer to them. You could then outline the different angles formed by the transversal in different colors so that all congruent angles are marked with the same color. This will help students more easily understand how a transversal can give them a lot of information around parallel lines.

Outlining the congruent angles with the same color will visually reinforce your teaching point on your board.